# THE 37th-9th INTERNATIONALRUDOLF ORTVAY PROBLEM SOLVING CONTEST IN PHYSICS 2006 

The Physics Students' Association of Eötvös University, Budapest, the Roland Eötvös Physical Society and the Hungarian Association of Physics Students proudly announce the $\mathbf{3 7 t h}$ - and for the ninth time international Rudolf Ortvay Problem Solving Contest in Physics, between 27 October 2006 and 6 November 2006.

Every university student from any country can participate in the Ortvay Contest. PhD students compete in a separate category. The contest is for individuals: solutions sent by groups of students are not accepted. The name, the university, the major, and the university year should be indicated on the solutions. Pseudonyms and passwords cannot be used: each contestant has to use his/her own name.

The problems can be downloaded from the webpages of the Ortvay Contest
http:/ /ortvay.elte.hu/
in Hungarian and English languages, in html, $\mathrm{LA}_{\mathrm{E}} \mathrm{X}$ and Postscript formats, from 12 o'clock (Central European Time, 10:00 GMT), Friday, 27 October 2006. The problems will also be distributed by local organizers at many universities outside of Hungary.

Despite all the efforts of the organizers, it may happen that some unclear points or misprints stay in the text. Therefore it is very useful to visit the webpage of the contest from time to time, as the corrections and/or modifications will appear there.

Each contestant can send solutions for up to 10 problems. For the solution of each problem 100 points can be given.
Any kind of reference material may be consulted; textbooks and articles of journals can be cited.
Each problem should be presented on (a) separate A4, or letter-sized sheet(s). The contestants are kindly asked to use only one side of each sheet. Solutions written by pencil or written on thin copy-paper will not be accepted as these cannot be faxed to the referees.

Computer programs appended to the solutions should be accompanied by detailed descriptions (what computer language it has been written in, how to use the program, which parameters can be set, what notations are used, how to interpret the output figures and graphs, etc.) They can be enclosed on floppy disks, or sent via email to the addresses below.

Solutions can be sent by mail, fax, or email (in $\mathrm{EA}_{\mathrm{E}} \mathrm{X}, \mathrm{T}_{\mathrm{E}} \mathrm{X}$, pdf or Postscript formats). Contestants are asked not to use very special $\mathrm{LA}_{\mathrm{E}} \mathrm{X}$ style files unless included in the sent file(s). Electronically submitted solutions should be accompanied-in a separate e-mail-by the contents and, if necessary, a description explaining how to open it.

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## Deadline for sending the solutions: 12 o'clock CET (11:00 GMT), 6 November 2006.

Contestants are asked to fill in the form available on our webpage after posting their solutions. It will be used for identification of contestants and their solutions. Without filling in the form, the organizers cannot accept the solutions! The form is available only on 6 to 7 November.

The contest will be evaluated separately for each university year, according to the total number of points. The referees reserve the right to withhold, to multiply or to share some prizes. Beyond the money prizes given for the first, second, and third places, honorable mentions and special prizes for the outstanding solutions of individual problems can be awarded. This is why it is worthwhile sending even one or two solutions.

The announcement of the results will take place on 7 December, 2006. The detailed results will be available on the webpage of the contest thereafter. Certificates and prizes will be sent by mail.

Wishing a successful contest to all our participants,

1. From the appendix of The Fanatics' Guide To the Galaxy:

Space tourists! Physicists! Meteorologists! Experts and gapers! Make the best of the never returning opportunity and visit (certainly via a trip organized by our travel agency, Cartesius Space Travels) Geminflecto, the 42th planet of the star Omikron in the constellation Rubberdog, the home of the galaxy-wide famous Zenithoal Rainbow. We certainly supply our guests with oxigen masks and spacesuits.
When at dawn the Omikron of Rubberdog pops up on the horizon of the bald, hoar-covered Great Plateau rising from the equatorial jungle of Geminflecto, our guests can be eye-wittnesses to unmached scenery. There is a 20 degrees wide, beautiful, bright rainbow constituting a full semicircle above their heads and passing exactly through the zenith. Unfortunately after a few minutes the hot rays of the Omikron end the phenomenon. Our guests can spend the forthcoming hours in an air conditioned refuge, and a few minutes before sunset they can venture upon going outside again to the slowly cooling plateau. The wonderful phenomenon, the Zenithoal Rainbow returns in the evening, at the moment of sunset.
We have to note that the Zenithoal Rainbow is accompanied by an even brighter rainbow in the western skyscape. Our customers are usually staring at the famous rainbow appearing at the zenith, therefore our travel agency is not in possession of accurate data about (the morning and evening position and width of) the Western Rainbow. Therefore we ask our physicist guests to inform us about these data should they get hold of them. We are also thankful if they give an explanation for the existence of the morning and evening Zenithoal and Western Rainbow. Our customers providing accurate information are by return of spaceship invited to a luxury trip to the planet Geminfelcto to inspect the Zenithoal Rainbow. (All right, we know that it would be easier to provide the data after the trip based on measurements performed on the spot, but we also know that this minor difficulty does not discourage our physicist customers from solving the problem.) Of course, our travel agency takes our inquisitive customers to other sceneries of the Galaxy as well. More information can be found in our new catalogue coming out in October, 2007.

(Gyula Dávid and József Cserti)



In his „Dialogo", published in 1632 (see the original figure), Galileo claims that if one drops an object from a tower on the rotating Earth, observed from an external inertail frame of reference it moves on a circular orbit crossing the center of the Earth until it hits the ground Was Galileo's claim true, what would the gravitational force law be? (According to Galileo the tower can be located anywhere on the Earth (except for the poles) and can be (in principle) of any height. When observed from an external inertial reference frame the rotation of the Earth gives an initial velocity to the dropped object. The top of the tower and the initial velocity vector defines a great circle of the Earth. This great circle is shown in the figure.)

## (Péter Horváthy)

3. Calculate the length of the day and the dusk on various latitudes during a revolution of the Earth around the Sun. Let $\gamma$ denote the latitude where the Sun culminates in zenith $\left(|\gamma| \leq \gamma_{\max }\right)$. Let $\gamma=0$ if the Sun shines perpendicularly to the equator, and $\gamma>0$ if it is spring or summer on the Northern Hemisphere. There is dusk on a given place if the Sun is below the horizon, but only by an angle not more than a given angle $\alpha$. Calculate the length of the day and the dusk at a given location as a function of the angle $\gamma$. Examine all locations on the Earth's surface. Obtain the value of $\gamma$ corresponding to the longest and shortest dusk for all latitudes, and specify the season it represents.
Use the following approximations:
a) Assume that the shape of the Earth is an exact sphere.
b) Assume that the Sun shines on exactly half of the Earth (i.e. it is observed above the horizon from half of the Earth).
c) Neglect the effect of the drawing in or lengthening of the days, i.e. take the length of the morning and the evening dusk the same, neglect their variation at a given value of $\gamma$.
d) Take $0^{\circ}<\gamma_{\max }<45^{\circ}$. Of course, in reality $\gamma_{\max }=23,5^{\circ}$.
e) Choose the dusk area realistically, i.e. choose $\alpha$ such that one of the poles is always in the dark. (In reality, the generally accepted value is $\alpha=18^{\circ}$.)
(Júlia Fekete and György Máté)
4. In a motorbike race we see riders driving their bikes with a uniform velocity in a long left turn, leaning left by an angle of approximately 60 degrees (from the vertical). At least how large should the fricition coefficient of the wheels and the road be in order to avoid skidding?
Just imagine a wheel falling off one of the bikes because of a technical problem. It is rolling furher on with the same leaning angle as before. Does the wheel roll off the road? If yes, does it leave the asphalt strip to the left or to the right?
(Assume realistic values for the missing data.)
5. According to one of Dirac's suggestions (cf. P. A. M. Dirac, A new basis for cosmology. Proc. Roy. Soc. A 165 (1938) 199-208) the gravitational coefficient in Newton's law is not a constant but is inversely proportional to time

$$
G=G(t)=G_{0} \frac{t_{0}}{t}
$$

where $t_{0}$ is a constant and $t$ is the age of the universe. Describe the motion of the planets in Dirac's modified theory.

## (Péter Horváthy)

6. A suspended string is stretched by its own weight. How does the initially resting string move if its suspension is released?
(István Groma)
7. Is it possible that the oscillations of a rocking horse accelerate while being damped?
(Ottó Haiman)
8. A group of theoretical physicists are wondering: What would Rutherford have got in his famous scattering experiment if the world were two dimensional?
Our experimental physicist colleagues take the trouble to examine the question with measurements (in our three dimensional world). They charge a very long and thin wire with radius $r$ such a way that the potential difference between the wire and a large, grounded flat metal sheet located at a distance $L$ from the wire is equal to $U$. Through a tiny hole on the metal sheet a monoenergetic ion beam is sent perpendicularly to the wire, on which (on whose electrostatic field) it scatters. The scattered particles are detected far away (at a distance much greater than $L$ ) from the wire.
How does the (appropriately defined) differential cross section measured in the scattering experiment depend on the scattering angle and the energy of the particles?
(Péter Gnädig)
9. How does Rutherford's formula, which describes the differential cross section of scattering on the central potential field $V(\mathbf{r})=\alpha / r$ of a fixed center, change when one considers relativistic particle velocities?
(Gyula Dávid)
10. Refine the description of the well known problem of scattering on a rigid sphere. Assume that the incoming projectile slightly indents the "almost rigid" sphere and is kicked back by the elastic force (with a very high string rate). (Here the "very high string rate" means that at the possible energies the depth of indentation is small compared to the radius of the sphere.) Find the differential and total cross section of the scattering using classical mechanical scattering theory. Which limit yields the well known results corresponding to a rigid sphere?
(Gyula Dávid)
11. Loránd Eötvös studied the value of the gravitational acceleration on the ice of Lake Balaton. He used the very accurate Eötvös pendulum for his measurements. He wanted to show whether there is an oil field under the lake or not. Is the measured gravitational acceleration $g$ greater or smaller compared to the case when there is no oil field under the lake? Let us model the oil field by a cylinder with height $h$ and radius $R$ whose center of mass is at a distance $d$ from the surface of the water. For simplicity assume that the base of the cylinder is parallel to the surface of the water.
a) Estimate, how many percent is the relative deviation of $g$ due to the oil field. Can this deviation be shown using the Eötvös pendulum? Use realistic parameters.
b) What can actually be measured using the Eötvös pendulum is the tensor $P_{i k}(\mathbf{r})=\frac{\partial^{2} U(\mathbf{r})}{\partial x_{i} \partial x_{k}}$ derived from the gravitational potential $U(\mathbf{r})$. (Here $x_{i}$ is the $i$ th component of the position vector $\mathbf{r}$.) Determine the tensor $P_{i k}(\mathbf{r})$ for a cylindrical oil field with negligible height ( $h \ll R$ ).
12. Asteroid 2001 Einstein is said to be not exactly spherical, but rather the shape of an egg. The simplest way to understand this shape is to imagine a perfect hemisphere of radius $R$ attached to half of an ellipsoid of revolution with a circular base of the same radius $R$ and a height, perpendicular to the base, of $(1+\epsilon) R$ with $\epsilon \ll 1$. Which of the following two actions requires more force: sliding the two halves of the planet upon each other, along the rotational axis, parallelly to it, or sliding the hemisphere on the half ellipsoid perpendicularly to the axis? How much force is needed in the two cases? (Assume that the mass distribution of the planet is homogeneous. In both cases, before trying to slide the halves of it, the planet is sawn in two pieces, thus one needs to act only against the gravitational force.)

(Előd Merse Gáspár)

13. We are trying to construct a model for the demonstration of drag. We install penduli of various resonant periods on an easily rolling cart. Initially, the cart and the penduli are at rest. Then we push the cart with a given initial velocity. The penduli start swinging, their phases soon become unordered, and the cart takes the velocity of the center of mass. Average out the effect of the motion of the penduli. What is the equation governing the motion of the cart if the density function $K(\omega)=\sum_{\alpha} \frac{\pi}{2} m_{\alpha} \omega_{\alpha}^{2} \delta\left(\omega-\omega_{\alpha}\right)$ characterizing the penduli is equal to a constant $(K(\omega)=\eta)$ ? The symbols $m_{\alpha}$ and $\omega_{\alpha}$ denote the mass and the angular frequency of pendulum number $\alpha$, respectively.

(Károly Vladár)

14. Sankt Gallen, Switzerland, at the time of the campaigns of Hungarians ... Explain how the ancient Hungarian reflex bow works. Assuming the sizes known from the archeological artefacts, the use of traditional materials and construction, and normal human anatomy, obtain the acceleration of the arrow until leaving the bow, and its final velocity. How far could one shoot with such a bow? What is the function of the limbs and the horns? Should they be light or heavy? Is it worth hanging heavy decorative objects on the limbs? Should the string be unstretchable or not?
(Szabolcs Márka)
15. The branches of tree trunks fallen into rapid rivers that hang out from the water often oscillate periodically in the plane given by the direction of the flow. Create a model for the phenomenon.
(Tamás Tél)
16. It is well known that one side of the Moon always faces the Earth, that is, the rotation of the Moon has synchronized with its orbitting around the Earth. The reason for this is the elastic deformation caused by the tidal forces, which is damped by interior friction of the material of the Moon. Can this phenomenon be understood by modelling the deformable planet orbitting around a fixed gravitational center by a system of two mass points connected with a damped string?
(Tamás Tél)
17. Due to surface tension, a coin made of aluminum can also "float" on the surface of water. If there are two such coins on the surface, they tend to get close to each other as if they attracted each other.
How does this attracting force depend on the distance of the two coins if the coins are relatively far from each other?
(We can also examine a "lower dimensional" version of the problem.)
(Péter Gnädig)
18. Given an infinitely deep swimming pool of area $S$, how much power should people swimming in random directions in the pool exert to excite water waves of typical height $h$ ?
(Péter Hantz)
19. A neutral metallic sphere is rotating uniformly about one of its diameters.
a) How does the charge distribution change inside the shpere and on its surface due to the rotation?
b) What is the induced magnetic field outside the sphere like?
c) Can such a mechanism explain the magnetic field of the Earth?
20. Two identical, closed, cylindrical containers are standing in a laboratory. They contain different quantities of the same type of liquid. Only its saturated vapor is present above the liquid, the pressure and temperature of which are identical in the two containers. When one starts to heat up the cylinders, the level of the liquid rises in one of them while falls in the other. How can this happen? What is needed in order to keep the level of the liquid unchanged while heating?
21. According to the measurements the observable 100-year tendency of the increase in the average temperature of the Earth is $a=0.7{ }^{0} C / 100$ years. This value is obtained by fitting a linear function $T_{i}=a \frac{i}{N}+b$ to the observed annual average temperature values ( $T_{i}, i=1,2, \ldots, N=100$ ), and $a$ gives the value of the tendency corresponding to $N$ years. The fitting is done using the method of least squares, that is, the obtained parameters $a$ and $b$ are the ones that minimize the expression $\sum_{i=1}^{N}\left[T_{i}-\left(a \frac{i}{N}+b\right)\right]^{2}$.
The debate about the warming is partly connected to the question whether the value $a$ is a statistical fluctuation. As a zeroth approach we can discuss the problem as follows. Assume that
a) There is no tendency. The annual average temperature values fluctuate around an average value $\bar{T}$.
b) Let the annual fluctuations be independent of each other.
c) Let the distribution of the annual fluctuations be Gaussian with a standard deviation $\sigma$.
d) Let $\sigma \approx 0.5{ }^{0} C$. This value can be obtained from the following estimate. The daily temperature fluctuation can be considered $\delta T \approx 5-10{ }^{0} C$, and since the annual average temperature is obtained as the average of 365 daily values, therefore $\sigma \approx(5-10){ }^{0} C / \sqrt{365} \approx 0.5{ }^{0} C$.
It is obvious that if we obtain $a>0.7^{0} C / 100$ years from the above considerations, then talking about a warming tendency does not make sense. It is also obvious that in the above problem the average of $a$ is $\bar{a}=0$, so by $a$ we mean $\sqrt{a^{2}}$. What is the value of $\sqrt{a^{2}}$ if $N=100$ and what is it if $N=10$ ?
(Zoltán Rácz)
22. The displacement field of a symmetric point defect in an elastic, isotropic medium is

$$
\mathbf{u}(\mathbf{r})=\frac{\Delta V}{4 \pi} \frac{\mathbf{r}}{|\mathbf{r}|^{3}},
$$

where $\Delta V$ is the deviation of the volume caused by the point defect. What is the interaction energy between two point defects?
(Géza Tichy)
23. The Einstein Express is moving at a relativistic constant velocity $V$ along straight, long, horizontal rails. Neglect the mass of the cars and model the wheels with disks of uniform mass distribution.
a) Find the position of the center of mass of a wheel in the reference frame fixed to the rails. Plot the position of the center of mass as a function of the velocity of the train.
b) The engine accelerates and exerts a horizontal force on the train. Some say that the center of mass of the wheels rise. How is it possible, since only horizontal force is acting on the wheels? What kind of force is doing the work of lifting?
(Gyula Dávid)
24. Zweistein's Theory of Hyper Relativity unifies Newtonian and Einsteinian mechanics, and in addition it opens new perspectives. According to the theory the connection between the energy and momentum of a point particle is given by an increasing function $E=f(p)$, where $p$ is the absolute value of the momentum (three-)vector $\mathbf{p}$. This connection is rather described using the Zweistein function $Z(E): p^{2} / 2=Z(E)$. If $Z_{0}(E)=m E$ then we obtain classical mechanics. The formula $Z_{1}(E)=\left(E^{2}-m^{2} c^{4}\right) / 2 c^{2}$ yields special relativity. And this is not all that Zweistein has for us...
Similarly to the classical case let the force $\mathbf{F}$ and acceleration a be the derivative of the (three-)momentum $\mathbf{p}$ and (three-)velocity $\mathbf{v}$ with respect to the time $t$ in a fixed reference frame, respectively. With the help of the Zweistein function $Z(E)$ write the equation of motion of the particle in a form similar to Newton's law $\mathbf{F}=\mathbf{M a}$. (Hint: consult Mr. Lagrange and Sir Hamilton for advice.) Examine the mathematical and physical properties of the generalized mass M. Find the values of the "transverse" and "longitudinal" masses. What is the Zweistein function like when the ratio of these two masses is independent of the energy of the particle? Investigate in detail the case of Zweistein functions of the form $Z_{N}(E)=\mu_{N}\left(E^{N}-E_{0}^{N}\right)$, where $N$ is a positive number and $\mu_{N}$ is a positive constant with appropriate dimension. Treat the case of $E_{0}=0$ separately.
25. Let us consider two types of circuits each consisting of 40 indentical light bulbs:
a) We create two cascades of 20 bulbs each then connect the two cascades in parallel.
b) We connect two bulbs in parallel then we cascade the such obtained 20 pairs.
The light bulbs can burn out independent of each other and according to a Poisson process, as a result of which they locally open the circuit. Examine two models: In the first model the Poisson intensity (that is, the parameter of the Poisson distribution) does not depend on the power delivered to the light bulb, while in the second one it is proportional to that. What is the distribution and expectation value of the time of the first burnout? What is the expectation value of the time of the second burnout? What is the expectation value of the time of the complete opening of the circuit? Which of the two circuits is "more stable"?

26. Prove that if an eigenstate of the Hamiltonian of a physical system is an intelligent state (that is, the product of the standard deviations of the position and the momentum is equal to the minimum value allowed by the uncertainty principle), than this system is the harmonic oscillator and the examined state is its ground state.

## (Géza Tichy)

27. The Hamiltonian of a free electron moving in the $(x, y)$ plane can be given by the expression $\mathbf{p}^{2} /(2 m)$, where $\mathbf{p}=\left(p_{x}, p_{y}\right)$. Examine the elastic scattering of plane waves on a spin dependent potential $V(\mathbf{r})$ induced by an impurity. Fix the spin quantizing axis perpendicular to the plane of the electrons. We know that the energy eigenfunction that describes the scattering of a plane wave incident from direction of angle $\varphi_{0}$ with a wave number $\mathbf{k}$ and an amplitude proportional to spinor $\gamma$ can be written at a point $P$ having polar coordinates $(r, \varphi)$ located very far from the scattering center in the form

$$
\psi(r, \varphi) \sim e^{i \mathbf{k r}} \gamma+\frac{e^{i k r}}{\sqrt{r}} f\left(\varphi, \varphi_{0}\right) \gamma
$$

where $f\left(\varphi, \varphi_{0}\right)$ is a $2 \times 2$ complex matrix, the so called scattering matrix, and $r=|\mathbf{r}|$ is the distance between point $P$ and the scattering center. Without the exact knowledge of the scattering potential determine what consequences are there for the scattering matrix if the spin dependent potential $V$ is invariant under
a) time reversal;
b) any rotation around the $z$ axis.
(András Pályi)
28. The heaviest known nucleus today is that of the super heavy element with atomic number 118 and mass number 294. It is produced in low energy nuclei scattering processes where it appears in very small quantities. However, the elapsed time between the formation and decay of every such nucleus can be measured very accurately (with an accuracy of 1 microsecond). Assume that the lifetime of 5 such nuclei could be measured in a long experiment, and the results were the following: $1.155 \mathrm{~ms}, 0.120 \mathrm{~ms}, 3.786 \mathrm{~ms}, 4.410 \mathrm{~ms}$ and 1.018 ms . What is the uncertainty of the halflife obtained from these results at a confidence level of $66 \%$ ? Determine the frequency distribution of the measured halflife.

## (Ákos Horváth)

29. Some radon is located in the air of a large room, and is in radioactive equilibrium with its daughter nuclides. With a suction pump we are pumping the air through a filter for several hours. The filtration efficiency is $100 \%$, that is, all the dust going through the pump sticks to the filter. (The daughter nuclides of Rn very well stick to the dust.) The speed of suction is $Q\left(\mathrm{~m}^{3} / \mathrm{s}\right) . Q$ is small, therefore during the several hours only a small fraction of the air goes through the suction pump. At time $t=0$ we turn off the suction pump and start to measure the alpha radiation of the filtering paper. Our detector is sensitive only to alpha radiation. What is the exact time dependence of the alpha intensity in the next hour if the activity of the Rn in the room is $1000 \mathrm{~Bq} / \mathrm{m}^{3}$ ? Give the formula and plot the time dependence. Try to write the formulas in the simplest possible and most practical form. The relevant part of the decay chain of radon: ${ }^{222} \mathrm{Rn} \rightarrow{ }^{218} \mathrm{Po} \rightarrow{ }^{214} \mathrm{~Pb} \rightarrow{ }^{214} \mathrm{Bi} \rightarrow{ }^{214} \mathrm{Po} \rightarrow{ }^{210} \mathrm{~Pb}$. The halflives of the isotopes listed above are 3.8 days, 3.05 minutes, 26.8 minutes, 19.9 minutes, 64 microseconds and 22 years, respectively.
30. Consider a spin that is placed in a homogeneous external magnetic field parallel to the $z$ axis. The magnitude of the field fluctuates in time according to white noise. Find the equation of motion of the density matrix using Ito calculus. Examine what happens to the system after verly long time if a superposed state is taken as initial condition.
(József Zsolt Bernád)
31. The Hamiltonian describing the motion of a (quasi)particle has the form

$$
H_{0}=\varepsilon(\mathbf{p}) \mathbb{1}+\boldsymbol{\Omega} \mathbf{S}
$$

where $\mathbf{p}$ is the momentum operator vector of the particle, $\varepsilon(\mathbf{p})$ is the usual dispersion relation (for example $\left.\varepsilon(\mathbf{p})=\mathbf{p}^{2} / 2 m\right), \mathbf{S}$ is the vector consisting of the irreducible spin matrices of index $j, \mathbb{1}$ is the $2 j+1$ dimensional unit matrix, $\boldsymbol{\Omega}=\mathbf{K p}$ and $\mathbf{K}$ is a given $3 \times 3$ constant matrix.
a) Derive the equation of motion for the $\mathbf{r}(t)$ position operator vector in Heisenberg picture, and solve the operator differential equation.
b) Show that in case of $j=1 / 2$ the most general (non-linear) Hamiltonian depending on $\mathbf{p}$ and $\mathbf{S}$ can be written in a form similar to the above one, where $\boldsymbol{\Omega}(\mathbf{p})$ is now a (generally non-linear) function of the momentum vector p. Solve the equation of motion of the operator vector $\mathbf{r}(t)$ in this case as well.
c) "Prepare" the initial state $\left|\psi_{0}\right\rangle$ of the system: $\left|\psi_{0}\right\rangle=\mathbf{P}|\psi\rangle$, where $|\psi\rangle$ is an arbitrary state vector and $\mathbf{P}$ is an appropriate projection operator. How does $\mathbf{P}$ have to be chosen if we want the expectation value of the position of the particle to move at a constant speed in a straight line as one would expect based on classical mechanics?
(Gyula Dávid and József Cserti)
32. Supersymmetric particles can be searched for at the LHC (Large Hadron Collider, starting in 2007) in decay chains

$$
\tilde{q}_{L} \rightarrow q \tilde{\chi}_{2}^{0} \rightarrow q \tilde{l} \tilde{l}^{+} \rightarrow q l^{+} l^{-} \tilde{\chi}_{1}^{0}
$$

A squark decays to a quark and neutralino then the neutralino further to an antilepton and slepton and finally the slepton decays to a lepton and a lighter neutralino. (In the supersymmetric standard model all known particles have a pair with different spin. The neutralinos are fermionic partners of standard neutral bosons and sleptons are bosonic partners of standard leptons.) In the experiment we would like to identify and measure the decay chain requiring: 2 isolated opposite charge lepton with $p_{t}>10 \mathrm{GeV}$, at least four jets, one with $p_{t}>100$ GeV and others with $p_{t}>50 \mathrm{GeV}$ and the missing transvers energy $E_{T}^{\text {miss }}>\max \left(100 \mathrm{GeV}, 0.2 M_{e f f}\right)$. Here $p_{t}$ is the momentum transverse to the beam pipe, the energy calculated from it is the transverse energy and $M_{e f f}=\sum_{i} p_{T, i}+E_{T}^{m i s s}$.
The mass of an $n$-particle system with four-momenta $p_{i}(i=1,2, \ldots n)$ is $m_{1 . . n}^{2}=\left(\sum_{i=1}^{n} p_{i}\right)^{2}$. Show that the mass of $q l^{+} l^{-}$falls between

$$
M_{q l l}^{\min } \simeq 271 \mathrm{GeV} \text { and } M_{q l l}^{\max }=\sqrt{\frac{\left(M_{\tilde{q}_{L}}^{2}-M_{\tilde{\chi}_{2}^{0}}^{2}\right)\left(M_{\tilde{\chi}_{2}^{0}}^{2}-M_{\tilde{\chi}_{1}^{0}}^{2}\right)}{M_{\tilde{\chi}_{2}^{0}}^{2}}} \simeq 552 \mathrm{GeV}
$$

(Gábor Cynolter)
33. Dr. Absoluto Zero, the dictator of Rubbercoast, the small equatorial country, who pretty much likes physics and (employs) physicists (about his goings-on see the previous years' Ortvay problems) was fairly pleased when he read Arthur C. Clarke's novel titled "The Fountains of Paradise". In this book the space stations orbiting around the Earth in stationary orbits, which are in connection with the Earth through space elevators, finally abut and form a rigid ring that is fixed to the Earth by "spokes" and create an ideal ground for settling down for the population of the overcrowded planet. What other material could be more appropriate for establishing this dream from than the super solid rubber nanotubes made of the juice of the gene manipulated and in Rubbercoast massively grown rubber palms. And how else could such a cosmic size construction be financed if not from the horribly large income of Rubbercoast resulting from rubber nanotubes. The thought was followed by action, and dr. Absoluto Zero, according to his habit has sent for his court main physicist again.
Ali Whyheknows was scared when he took the plans of the gigantic establishment from the hands of the dictator. He tried to draw dr. Absoluto Zero's attention to the minor fact that the plan had been rotated by 90 degrees compared to its original orientation, but this issue was made a short work of being neglected by the dictator. Anyhow the distinction of left and right together with up and down has always been a prickly political question in Rubbercoast, therefore the main physicist did not labor the point.
So the Huge Wheel (certainly named after dr. Absoluto Zero) has been built. True that is was not built along the equator but along the meridian going through Port Goomy, the capital of Rubbercoast. As a result the
previously registered settlers did not rush for the empty sites. They referred to such ridiculous reasons as the ground is very steep and it is difficult to build houses on it. The officials of Rubbercoast then always pinned down that this is wrong, since the circumference of the Huge Wheel is parallel to the surface of the Earth.
a) Help settling down the argument. Determine and illustrate graphically what kind of mountain terrain the Huge Wheel is equivalent to.
b) After waiting several years in vain Ali Whyheknows suggested a new way of making use of the Huge Wheel: its smooth surface, which is free of settlers, is an ideal location for the Formula 42 around-the-Earth spacecar racing. Examine the question in case of spacecars both with and without an engine (initially the cars are at rest). Where should the location of the start point be? What is the maximum achievable speed? How long does an around-the-Earth run take? What should the designers of the tyres be careful about? etc. Don't forget: the super solid rubber nanotubes made of the juice of the gene manipulated rubber palms grown in Rubbercoast can take everything.
c) Suggest more aims (in connection with sports, culture, tourism, traffic etc.) for the use of the Huge Wheel lying (excuse me, rotating) waste.

